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On the various type of Mappings in *S*-Menger space with application

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Abstract: In the present paper, we investigate various mapping like compatible mappings, weakly compatible mappings, semi-compatible mappings, and mappings satisfying the property (E.A.). By employing these concepts, a common fixed-point theorem is established in the framework of *S*-Menger spaces. Our result extends the corresponding theorem of Aalam et al. [1] and generalizes it to the setting of *S*-Menger spaces. Furthermore, examples are provided to demonstrate the applicability and effectiveness of the established result.

Keywords: *S*-Menger Space, Fixed Point, Compatible Maps, Weak Compatible Maps, Semi-compatible Maps, Compatible Maps, The Property (E.A.).

MSC: 47H10, 54H25.

1. Introduction

In 1942, Menger [7] introduced the concept of probabilistic metric spaces, where the distance between two points is represented by a distribution function rather than a non-negative real number. This idea initiated a new direction in the study of generalized metric structures and their applications. Later, Gähler [5,6] introduced the notion of 2-metric spaces as a generalization of ordinary metric spaces, which further stimulated the development of generalized distance spaces.

In 1992, Dhage [3] introduced the concept of D-metric spaces and investigated several fixed-point results in this setting. Subsequently, Dhage [4] studied the topological structure of generalized metric spaces and established various fundamental properties. However, some limitations associated with D-metric spaces were later identified. To overcome these difficulties, Mustafa and Sims [8] introduced G-metric spaces, providing a more suitable framework for the study of fixed-point theory and related topics.

Motivated by the development of generalized metric spaces, Sedghi, Shobe, and Aliouche [8] introduced the concept of S-metric spaces in 2012. The class of S-metric spaces extends and unifies several existing generalized metric structures and has become an active area of research in fixed-point theory. The rich topological and analytical properties of S-metric spaces have enabled researchers to establish numerous fixed-point results under different contractive and compatibility conditions.

On the other hand, the theory of probabilistic metric spaces has continued to evolve since the pioneering work of Menger [7]. The combination of probabilistic concepts with generalized metric structures has provided powerful tools for studying uncertainty and randomness in nonlinear analysis. Recently, Sarkar, Das, and Pramanik [10] introduced the notion of *S*-Menger spaces and established fixed-point results of Banach and Kannan type in this framework. Their work demonstrated that *S*-Menger spaces provide a natural extension of both Menger spaces and S-metric spaces and offer a fruitful setting for further investigations.

The study of various forms of compatibility among mappings plays an important role in common fixed-point theory. Concepts such as compatible mappings, weakly compatible mappings, semi-compatible mappings, and the property (E.A.) have been extensively used to obtain generalized fixed-point theorems under weaker assumptions. These notions have significantly broadened the scope of fixed-point theory and its applications in generalized spaces.

In the present paper, we investigate several mapping properties in the setting of *S*-Menger spaces. We introduce and study compatible mappings, weakly compatible mappings, semi-compatible mappings, and mappings satisfying the property (E.A.) in *S*-Menger spaces. Various auxiliary examples are established to illustrate the relationships among these concepts. As an application of the developed theory, we obtain a common fixed-point theorem in *S*-Menger spaces which extends and generalizes the corresponding fixed-point theorem of Aalam, Kumar, and Pant [1]. Furthermore examples are provided to demonstrate the applicability and effectiveness of the obtained result.

2. Preliminaries

We recall the following definitions of *S*-Menger spaces.

Definition 2.1.[2] A map $*: X^3 \to X$ is called continuous t-norm if it satisfies the following conditions:

(i) $*(a,1,1) = a$, *(0,0,0) =0;

(ii) $*(a,b,c) = *(a,c,b,) = *(b,c,a)$;

(iii) $*(a_1,b_1,c_1) \geq *(a_2,b_2,c_2)$ for $a_1 \geq a_2, b_1 \geq b_2, c_1 \geq c_2$.

Examples of t −norm are

(1): $x * y * z = x.y.z$ and

(2): $x * y * z = \min\{x.y.z\}$ minimum t −norm

Definition 2.2. ([10] S-Menger space)

The 3-tuple $(X, S, *)$ is said to be S-Menger space if X is a non-empty set, S is a function defined on X^3 to the set of distribution function and $*$ is a continuous third order t-norm such that the following conditions are satisfied:

(i) $S_{(x,y,z)}(0) = 0$ for all $x, y, z \in X$
(ii) $S_{(x,x,y)}(t) < 1$ for $t > 0$ with $x \neq y$,
(iii) $S_{(x,y,z)}(t) = 1$ for all $t > 0$, if and only if $x = y = z$,
(iv) $S_{(x,y,z)}(t) \geq * \left(S_{(x,x,a)}(t_1), S_{(y,y,a)}(t_2), S_{(z,z,a)}(t_3)\right)$,
where $t = t_1 + t_2 + t_3$ and $t, t_1, t_2, t_3 > 0$ for all $x, y, z, a \in X$

Definition 2.3. [10] The S −Menger space is called symmetric if $S_{(x,x,y)}(t) = S_{(y,y,x)}(t)$

Definition 2.4. [10] Let $(X, S, *)$ be a symmetric S −Menger space then a sequence $\{x_n\} \in X$ is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} S_{(x_n,x_n,x)}(t) = 1$ for all $t > 0$.

Definition 2.5. [10] Let $(X, S, *)$ be a symmetric S − Menger space then a sequence $\{x_n\} \in X$ is called Cauchy sequence if $x \in X$ if $\lim_{n\to\infty} S_{(x_n,x_n,x_{n+s})}(t) = 1$ for all $t, s > 0$.

Lemma 2.6.[10] Let $(X, S, *)$ be an S-Menger space with continuous third order t-norm. Then $S_{(x,x,y)}(t)$ is non-decreasing with respect to t, for all $x, y \in X$

Main Results:

Definition 3.1. Let g and f maps from an S-Menger space *(X, S, ∗)* into itself. The maps g and f are said to be compatible, if for all $t > 0$.

$$\lim_{n\to\infty} S_{(gfx_n,gfx_n,fgx_n)}(t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = z$ for some $z \in X$.

Example 3.2. Let $X = [2,20]$. For each $t \in (0, \infty)$ and for all $x, y, z \in X$, define

$$S_{(x,y,z)}(t) = \begin{cases} 0, & t = 0 \\ \frac{t}{t+|x-z|+|y-z|}, & t \geq 0 \end{cases}$$

Clearly $(X, S, *)$ is an S-Menger space, where $*$ is defined by $a * b * c = abc$. Let g and f be self-maps of X defined as

$$g(x) = \begin{cases} 2, & if\ x = 2 \text{ or } x > 5 \\ 6, & if\ 2 < x \leq 5 \end{cases} \quad f(x) = \begin{cases} 2 & if\ x = 2\ or\ x > 5 \\ 12 & if\ 2 < x \leq 5 \\ \frac{x+1}{3} & if\ x > 5. \end{cases}$$

Let sequence $\{x_n\}$ be defined as $x_n = 5 + 1/n$, $n \geq 1$ then we have

$\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = 2$. Hence, g and f satisfy the property (E.A.). Also, $\lim_{n\to\infty} S_{(gfx_n,gfx_n,fgx_n)}(t) = \frac{t}{t+|2-2|+|2-2|} = \frac{t}{t+0} = 1$ This shows that g and f are compatible.

Definition 3.3. Let *g* and *f* be maps from an *S*-Menger space *(X, S, ∗)* into itself. The maps are said to be weakly compatible, if they commute at their coincidence points, that is, *gz* = *f z* implies that *gf z* = *fgz*.

Definition 3.4. Let *g* and *f* maps from an *S*-Menger space *(X, S, ∗)* into itself. The maps *g* and *f* are said to be semi-compatible, if for all *t* > 0.

$$\lim_{n\to\infty} S_{(gfx_n,gfx_n,fgz)}(t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = z$ for some $z \in X$.

Note that the semi-compatibility of the pair *(g, f)*, need not imply the semi-compatibility of *(f, g)*.

The following is an example of a pair of self-maps *(g, f)* which is compatible but not semi-compatible. Further, it is also seen that the semi-compatibility of the pair *(g, f)* need not imply the semi-compatibility of *(f, g)*.

Example 3.5. Let $X = [0,1]$ and be the *S*-Menger space with

$$S_{(x,y,z)}(t) = \begin{cases} e^{[\frac{|x-z|+|y-z|}{t}]^{-1}}, & t > 0 \\ 0 & t = 0 \end{cases} \text{ for all } x, y, z \in X, t > 0.$$

Define self-maps *S* as follow

$$g(x) = \begin{cases} x, & \text{when } 0 \le x < \frac{1}{2} \\ 1, & \text{when } \quad x \ge \frac{1}{2} \end{cases}$$

Let *I* be the identity map on X and $x_n = \frac{1}{2} - \frac{1}{n}$. Then, $\{Ix_n\} = \{x_n\} \to \frac{1}{2}$ and $\{gx_n\} = \{x_n\} \to \frac{1}{2}$. Thus, $\{Igx_n\} = gx_n\} = \frac{1}{2} \neq g\{\frac{1}{2}\}$. Hence (*Ig*) is not semi-compatible.

Again as (*I*, *g*) is commuting, it is compatible. Further, for any sequence $\{x_n\}$ in *X* such that $\{x_n\} \to x$ and $\{gx_n\} \to x$, we have $\{gIx_n\} = \{gx_n\} \to x = I\,x$. Hence (*gI*) is always semi-compatible

Remark 3.6. The above example gives an important aspect of semi-compatibility as the pair of self-maps (*Ig*) is commuting; hence it is weakly commuting, compatible and weak compatible yet it is not semi-compatible. Further, it is to be noted that the pair (*g*, *I*) is semi-compatible but (*I*, *g*) is not semi-compatible here. The following is an example of a pair of self-maps (A, g) which is semi-compatible but not compatible.

Example 3.7. Let $X = [0,2]$ and *(X, S, ∗)* be an *S*-Menger space, where the definition of * and *g* are same as defined in example 4.2. Define self-maps *A* and *g* on *X* as follows.

$$A(x) = \begin{cases} 2, & if\ 0 \le x \le 1 \\ \frac{x}{2}, & if\ 1 < x \le 2 \end{cases} \qquad g(x) = \begin{cases} 2 & , if\ x = 1 \\ \frac{x+3}{5} & , \text{otherwise} \end{cases}$$

and $x_n = 2 - \frac{1}{2n}$,$n \geq 1$. Then we have $g(1) = A(1) = 2$ and $g(2) = A(2) = 1$. Also $gA(1) = Ag(1) = 1$ and $gA(2) = Ag(2) = 2$. Thus (A, g) is weak compatible. Again $Ax_n = 1 - \frac{1}{4n}$,$gx_n = 1 - \frac{1}{10n}$. Thus $Ax_n \to 1, gx_n \to 1$. Hence $u = 1$.

Further, $gAx_n = \frac{4}{5} - \frac{1}{20n}$,$Agx_n = 2$.

Now $\lim_{n\to\infty} S_{(Agx_n, Agx_n, gu)}(t) = \lim_{n\to\infty} S_{(2,2,2)}(t) = 1$

$$\lim_{n\to\infty} S_{(Agx_n, Agx_n, gAx_n)}(t) = \lim_{n\to\infty} S_{\left(2,2,\frac{4}{5}-\frac{1}{20n}\right)}(t)$$

$$= \frac{t}{t + \left|2 - \frac{4}{5}\right| + \left|2 - \frac{4}{5}\right|}$$

$$= \frac{t}{t + \frac{12}{5}} < 1 \qquad \forall\, t > 0.$$

Hence (A, g) is semi-compatible but it is not compatible.

Definition 3.8. Let g and f be two self-maps of an S-Menger space $(X, S, *)$. We say that g and f satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = z$ for some $z \in X$.

Remark 3.9. Note that the weakly compatible and property (E.A.) are independent to each other. See the following example

Example 3.10. Let $(X, S, *)$ be an S-Menger space where $X = [0,1]$ and S is defined as in 3.2. Defined $g, f : X \to X$ by

$g(x) = 1 - x$, if $x \in [0, \frac{1}{2}]$ and $g(x) = 0$, if $x \in (\frac{1}{2}, 1]$

$f(x) = \frac{1}{2}$, if $x \in [0, \frac{1}{2}]$ and $f(x) = \frac{3}{4}$, if $x \in (\frac{1}{2}, 1]$

Then, for the sequence $\{x_n\} = \left\{\frac{1}{2} - \frac{1}{n}\right\}, n \geq 2$. we have

$$\lim_{n\to\infty} g\left(\frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{n}\right) = \frac{1}{2} = \lim_{n\to\infty} f\left(\frac{1}{2} - \frac{1}{n}\right)$$

Thus, the pair (g, f) satisfies property *(E.A)*. Further, g and f are weakly compatible since $x = \frac{1}{2}$ is their unique coincidence point and $gf\left(\frac{1}{2}\right) = g\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right) = fg\left(\frac{1}{2}\right)$.We further observe that

$$\lim_{n\to\infty} S(gf\left(\frac{1}{2} - \frac{1}{n}\right), gf\left(\frac{1}{2} - \frac{1}{n}\right), fg\left(\frac{1}{2} - \frac{1}{n}\right)) = \frac{t}{t + 2\left|gf\left(\frac{1}{2} - \frac{1}{n}\right) - fg\left(\frac{1}{2} - \frac{1}{n}\right)\right|}$$

$$= \frac{t}{t + 2\left|\frac{1}{2} - \frac{3}{4}\right|} = \frac{t}{t + \frac{1}{2}} \neq 1$$

showing that the pair (g, f) is noncompatible.

A Class of Implicit Relation: Let $\emptyset$ be the set of all real continuous functions $\phi : [\mathbb{R}^+]^4 \to \mathbb{R}$, nondecreasing in first argument and satisfying the following conditions.

(i) For $u, v \geq 0, \phi(u, v, v, u) \geq 0$ or $\phi(u, v, u, v) \geq 0$ implies $u \geq v$ …. (A)

(ii) $\phi(u, v, 1,1) \geq 0$ *implies that* $u \geq 1$ ………….. (B)

Example 3.11. (i) Define $\phi(t_1, t_2, t_3, t_4) = 15t_1 - 13t_2 + 5t_3 - 7t_4$. Then $\phi \in \emptyset$

(ii) Define $\phi(t_1, t_2, t_3, t_4) = 14t_1 - 12t_2 + 6t_3 - 8t_4$. Then $\phi \in \emptyset$

Application of compatible maps and (E.A) property: As an application of weak compatible maps and the property (E.A), we prove the fixed-point theorem of Irshad Aalam et.al [1] in *S*-Menger space.

Theorem 3.12: Let L, M, N and O be self-maps of an *S*-Menger space *(X, S, ∗)* satisfying the following conditions:

$L(X) \subseteq O(X), M(X) \subseteq N(X)$; ………… (3.12.1)

(L, N) *and* (M, O) are weakly compatible pairs ………………. (3.12.2)

(L, N) or (M, O) satisfies the property (E.A) ; ………………….(3.12.3)

For some $\phi \in \emptyset$, there exist $k \in (0,1)$ such that for all $x, y \in X, t > 0$

$\phi(S_{(Lx,Lx,My)}(kt), S_{(Nx,Nx,Oy)}(t), S_{(Lx,Lx,Nx)}(t), S_{(Ly,Ly,Oy)}(t)) \geq 0$….. (3.12.4)

If the range of one of the maps *L*, *M*, *N* or *O* is a complete subspace of *X*, then *L*, *M*, *N* and *O* have a unique common fixed point in *X*.

Proof: If the pair (M, O) satisfies the property (E.A) then there exist a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Mx_n \to z$ and $\lim_{n\to\infty} Ox_n \to z$ for some $z \in X$ as $n \to \infty$.

Since $M(X) \subseteq N(X)$,there exist in X a sequence $\{y_n\}$such that $Mx_n \subseteq Nx_n$.

Hence $Ny_n \to z$ as $n \to \infty$.

Now we claim that $Ly_n \to z$ as $n \to \infty$.Suppose $Ly_n \to w(\neq z) \in X$, then by (3.12.4), we have

$$\phi(S_{(Ly_n,Ly_n,Mx_n)}(kt), S_{(Ny_n,Ny_n,Ox_n)}(t), S_{(Ly_n,Ly_n,Ny_n)}(t), S_{(Mx_n,Mx_n,Ox_n)}(t)) \geq 0$$

that is,

$$\phi(S_{(Ly_n,Ly_n,Mx_n)}(kt), S_{(Mx_n,Mx_n,Ox_n)}(t), S_{(Ly_n,Ly_n,Mx_n)}(t), S_{(Mx_n,Mx_n,Ox_n)}(t)) \geq 0$$

As ϕ is nondecreasing in the first argument, we have

$$\phi(S_{(Ly_n,Ly_n,Mx_n)}(t), S_{(Mx_n,Mx_n,Ox_n)}(t), S_{(Ly_n,Ly_n,Mx_n)}(t), S_{(Mx_n,Mx_n,Ox_n)}(t)) \geq 0$$

Using (A), we get $S_{(Ly_n,Ly_n,Mx_n)}(t) \geq S_{(Mx_n,Mx_n,Ox_n)}(t)$. Letting $n \to \infty$,$S_{(w,w,z)}(t) \geq 1$ for all $t > 0$. Hence $S_{(w,w,z)}(t) = 1$ Thus $w = z$.

This shows that $Ly_n \to z$ as $n \to \infty$.

suppose that $N(X)$ is complete subspace of X. Then $z = Nu$ for some $u \in X$. Subsequently, we have $Ly_n \to Nu$, $Mx_n \to Nu$, $Ox_n \to Nu$ and $Ny_n \to Nu$ as $n \to \infty$.

By (3.12.4), we have

$$\phi(S_{(Lu,Lu,Mx_n)}(kt), S_{(Nu,Nu,Ox_n)}(t), S_{(Lu,Lu,Nu)}(t), S_{(Mx_n,Mx_n,Ox_n)}(t)) \geq 0$$

Letting $n \to \infty$

$$\phi(S_{(Lu,Lu,Nu)}(kt), 1, S_{(Lu,Lu,Nu)}(t), 1) \geq 0$$

As ϕ is nondecreasing in the first argument, we have

$$\phi(S_{(Lu,Lu,Nu)}(t), 1, S_{(Lu,Lu,Nu)}(t), 1) \geq 0$$

By using (A), we get $S_{(Lu,Lu,Nu)}(t) \geq 1$ for all $t > 0$. Hence, $S_{(Lu,Lu,Nu)}(t) = 1$.

Thus,$Lu = Nu$.

The weak compatibility of L and N implies that $LNu = NLu$ then $LLu = LNu = NLu = NNu$.

On the other hand, since $L(X) \subseteq O(X)$, there exists a $v \in X$ such that $Lu = Ov$. we show that $Ov = Mv$. By (3.12.4), we have

$$\phi(S_{(Lu,Lu,Mv)}(kt), S_{(Nu,Nu,Ov)}(t), S_{(Lu,Lu,Nu)}(t), S_{(Mv,Mv,Ov)}(t)) \geq 0$$

that is,

$$\phi(S_{(Ov,Ov,Mv)}(kt), 1,1, S_{(Mv,Mv,Ov)}(t)) \geq 0$$

As ϕ is nondecreasing in the first argument, we have

$\phi(S_{(Ov,Ov,Mv)}(kt), 1,1, S_{(Mv,Mv,Ov)}(t)) \geq 0$ [As $S_{(x,x,y)}(t) = S_{(y,y,x)}(t)$]

Using (A), we get $S_{(Ov,Ov,Mv)}(t) \geq 1$ for all $t > 0$. Hence, $S_{(Ov,Ov,Mv)}(t) = 1$.

Thus, $Mv = Ov$.

This implies $Lu = Nu = Ov = Mv$. The weak compatibility of M and O implies that $MOv = OMv$ and then $OOv = OMv = MOv = MMv$.

Now, we will show that Lu is a common fixed point of L, M, N and O.

In view of (3.12.4) it follows

$$\phi(S_{(LLu,LLu,Mv)}(kt), S_{(NLu,NLu,Ov)}(t), S_{(LLu,LLu,NLu)}(t), S_{(Mv,Mv,Ov)}(t)) \geq 0$$

that is,

$$\phi(S_{(LLu,LLu,Lu)}(kt), S_{(LLu,LLu,Lu)}(t), 1,1) \geq 0$$

As ϕ is nondecreasing in the first argument, we have

$$\phi(S_{(LLu,LLu,Lu)}(t), S_{(LLu,LLu,Lu)}(t), 1,1) \geq 0$$

Using (B), we get $S_{(LLu,LLu,Lu)}(t) \geq 1$ for all $t > 0$. Hence $S_{(LLu,LLu,Lu)}(t) = 1$.

Thus $LLu = Lu$.

Therefore, $Lu = LLu = NLu$ and Lu is a common fixed point of L and N. Similarly, we prove that Mv is a common fixed point of M and O. Since $Lu = Mv$, we conclude that Lu is a common fixed point of L, M, N and O . The proof is similar when $O(X)$ is assumed to be a complete subspace of X. The cases in which $L(X)$ or $M(X)$ is a complete subspace of X are similar to the cases in which $O(X)$ or $N(X)$ respectively, is complete since $L(X) \subseteq O(X)$, $M(X) \subseteq N(X)$.

$Lu = Mu = Ou = Nu = u$ and $Lv = Mv = Nv = Ov = v$ then (3.12.4) gives

$$\phi(S_{(Lu,Lu,Mv)}(kt), S_{(Nu,Nu,Ov)}(t), S_{(Lu,Lu,Nu)}(t), S_{(Mv,Mv,Ov)}(t)) \geq 0$$

that is ,

$$\phi(S_{(u,u,v)}(kt), S_{(u,u,v)}(t), 1,1) \geq 0$$

sing (B), we get $S_{(u,u,v)}(t) \geq$ for all t > 0. Hence, $S_{(u,u,v)}(t) = 1$. Thus, $u = v$.

Therefore, the common fixed point is unique.

The following example illustrates our result.

Example 3.13. Let *(X, S, ∗)* be an *S*-Menger space as defined in example 3.2.

Defined $L, M, N, O: X \to X$ by

$$L(x) = \begin{cases} 2, & \text{when } x = 2 \\ 1, & \text{when } 2 < x < 20 \end{cases}$$

$$M(x) = \begin{cases} 2, & \text{when } x = 2 \\ 7, & \text{when } 2 < x < 20 \end{cases}$$

$$N(x) = \begin{cases} 2 & \text{when } x = 2 \\ 6 & \text{when } 2 < x \leq 10 \\ (x-7) & \text{when } 10 < x < 20 \end{cases}$$

$$O(x) = \begin{cases} 2 & \text{when } x = 2 \\ 3 & \text{when } 2 < x \leq 10 \\ (x-3) & \text{when } 10 < x < 20 \end{cases}$$

Then *L*, *M*, *N* and *O* satisfy all the conditions of theorem with $k \in (0, 1)$ and have a unique common fixed-point $x = 2$. Clearly *(L, N)* and *(M, O)* are weakly compatible since they commute at their coincidence points. Let sequence $\{x_n\}$ be defined as $x_n = 10 + \frac{1}{n}$, $n \geq 1$, then we have $\lim_{n\to\infty} M\, x_n$=$\lim_{n\to\infty} M\, x_n$=7. Hence, *M* and *O* satisfy the property *(E.A.)*.

Conclusion: In this paper, we examined several important classes of mappings, including compatible mappings, weakly compatible mappings, semi-compatible mappings, and mappings satisfying the property *(E.A.)*, within the framework of *S*-Menger spaces. By utilizing these concepts, we established a common fixed-point theorem that extends and generalizes the corresponding result of Aalam et al. [1] to the setting of *S*-Menger spaces. The validity and applicability of the obtained theorem were further illustrated through appropriate examples. The results presented in this work enrich the existing literature on fixed-point theory in generalized probabilistic metric spaces and open new avenues for further research in the theory and applications of *S*-Menger spaces.

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