

THE USE OF TWO-SAMPLE t -TEST IN THE REAL DATA

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Abstract

The t-test is among the most frequent uses of statistics for hypothesis testing and comparing means of populations. The t-test was developed and improved by statistic pioneers and has been extensively used ever since for various purposes and in different contexts of research. The t-test for two samples is applied to check whether two independent populations have equal means or not. The real Infant Mortality Rate (IMR) data is taken into consideration for testing purposes to apply the significance of the t-test. The descriptive statistical analysis step begins with calculating descriptive statistics, and then null and alternative hypotheses are stated. The pooled variance method is used to calculate the test statistic value and then the t-value is determined based on the chosen significance level. The outcome clearly reveals how the two-sample t-test can be efficiently used to figure out whether there is significance or not among two populations using real data.

1. Introduction

T-test is amongst the most widely used statistical tools for hypothesis and comparison of means of populations. T-test has had historical significance with key contributions from William Gosset and Karl Pearson and R. A. Fisher. T-test has been in use since its origin and is widely considered to be an important part of statistics given its ease and efficiency in application, even in conditions where sample size is relatively small and variances are not known.

Test of significance is a statistical procedure where there is evaluation of claims regarding a population parameter, like mean value. Hypothesis about this claim is evaluated by means of sample data, and the purpose of hypothesis testing is to reveal if there is enough difference between sample statistics in order to attribute it to random sampling or whether there is some difference between population parameters.

Of the different types of t-tests, the two sample t-test is extensively used when the intention is to compare the means of two independent populations. The two sample t-test is preferentially employed in real-life applications where the samples are independently selected, and the parent distributions are assumed to be normally distributed. In the two sample t-test, the null hypothesis is the equality of the means, while the alternative hypothesis states the means to be different.

A t-test result can be interpreted on the basis of the test statistics and the corresponding p-values, which represent the evidence against the null hypothesis. If the calculated T-value exceeded the critical value for a certain significance level, then the null hypothesis would be rejected, and it would be concluded that there was a statistically significant difference between the two population means.

This paper will demonstrate the practical application of the two-sample t-test based on real data obtained in practice. Official data on the Infant Mortality Rate (IMR) will be used exclusively in this paper for illustration purposes of how the test can be applied in practice. The main purpose of this research is to appeal to the importance of the two-sample t-test within the process of real data analysis and the assessment of the statistical significance of the differences found in two populations.

2. Literature Review

The t-test has a history that dates back to 1908 when it was discovered by William Gosset but written under the pen name "Student." It can be defined as a statistical test that has a fitting distribution if and whenever the population from which the sampling is done has a normal distribution. During this time, the t-test has emerged as the statistical method that is used on a routine basis in testing hypotheses in numerous fields due to its efficiency and versatility. Credit for advancements in this method can be given to R.A. Fisher, Karl Pearson, and William Gosset.

Initially, this test was further developed to create the two-sample t-test by Snedecor and Cochran. In this way, this test was able to compare the means of two independent populations. The primary statistical problem faced by Gosset was concerned with how good a sample standard deviation was as an estimator of a population standard deviation, especially where smaller sample sizes were being utilized. In this way, there was a concern regarding how significant certain differences were.

A typical use of the two-sample t-test is the comparison of a new process, treatment, or condition against an existing process, treatment, or condition to determine which one is superior. As a result of a typical experiment design that involves comparing the means of two samples, the two-sample t-test is algebraically identical to a one-way analysis of variance with two levels. McDonald pointed out that the two-sample t-test needs two variables: a variable defining membership and a variable representing the measure of interest.

According to Peck, et al., two samples will be regarded as independent if the observations in one sample do not affect the observations in the second sample. In such a situation, the two-sample t-test procedure can be used to compare the means of the two groups. But if, for instance, the observations are matched or paired in a special way, then the paired t-test procedure should be employed. The t-test procedure can be viewed as a powerful statistical procedure for testing for differences between means, but with moderate power, since the effectiveness of the procedure is dependent on the size of the sample. The bigger the sample size, the smaller the t-value required for statistical significance.

Although it is a useful statistical tool, there are instances of misuse of the t-test in research. Foster and Gerald pointed out the problem of coming up with incorrect inferences due to repeated use of the t-test on the same data. In fact, the two-sample t-test should be used with careful consideration of the assumptions.

3. Derivation of the Two-Sample *t*-Test

Let

$X_{11}, X_{12}, \dots, X_{1n_1}$ and $X_{21}, X_{22}, \dots, X_{2n_2}$

be two independent samples drawn from two normal populations with respective means μ_1 and μ_2 , and a common population variance σ^2 .

The unbiased estimators of the population means μ_1 and μ_2 are given by the sample means:

$$\bar{X}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} X_{ji}, j = 1, 2 \quad (1)$$

The unbiased estimators of the population variance for each sample are defined as:

$$s_j^2 = \frac{1}{n_j - 1} \sum_{i=1}^{n_j} (X_{ji} - \bar{X}_j)^2, j = 1, 2 \quad (2)$$

Assuming that the two populations have equal variances, a pooled estimator of the common variance σ^2 is obtained by combining the sample variances as follows:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \quad (3)$$

Using this pooled variance estimator, the test statistic for comparing the two-population means is constructed as:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad (4)$$

The resulting statistic follows a t -distribution with $n_1 + n_2 - 2$ degrees of freedom. This test statistic is used to determine whether the observed difference between the two-sample means is statistically significant. If the calculated t -value exceeds the critical value at a chosen level of significance, the null hypothesis of equal population means is rejected.

4. Application of the Two-Sample t -Test

In this section, the two-sample t -test is applied to real data in order to examine whether there is a statistically significant difference between the Infant Mortality Rates (IMR) of selected developing and developed countries. The IMR data are obtained from official international health statistics and are expressed as the number of infant deaths per 1,000 live births.

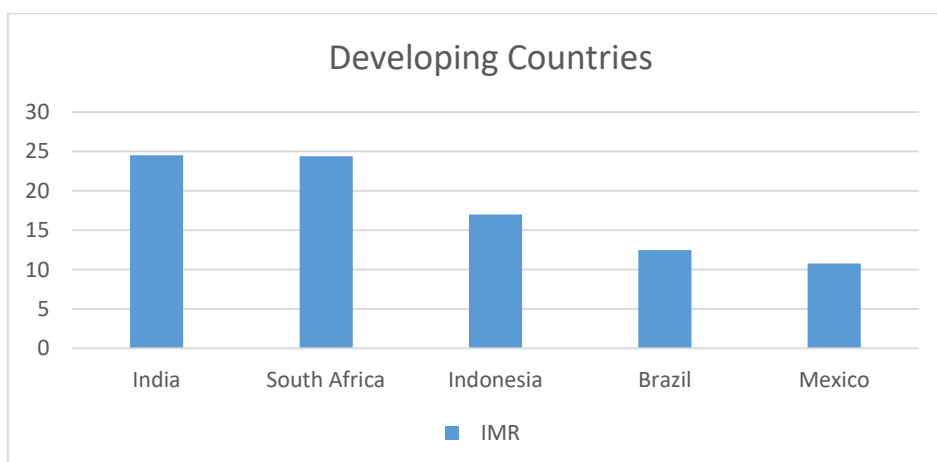
IMR Data

The data used in this study are presented in Table 1.

Table 1. IMR values of selected developing and developed countries

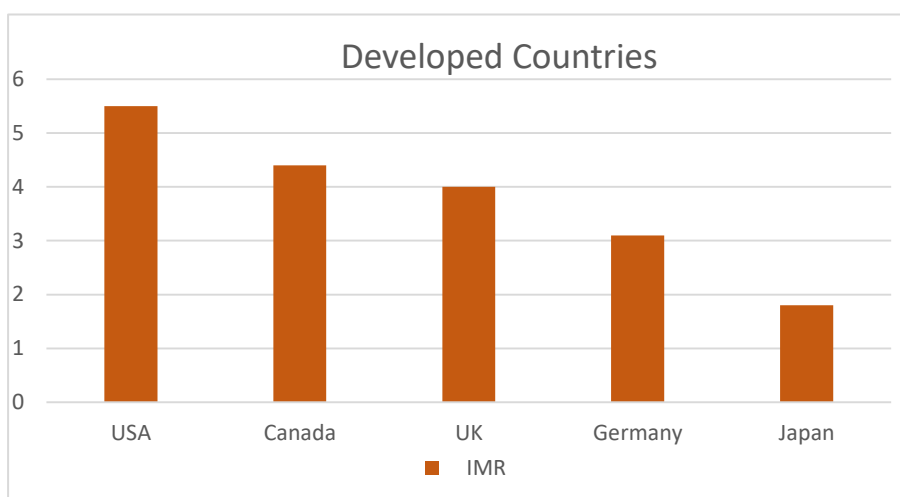
Developing Countries IMR

Developing Countries	IMR
India	24.5
South Africa	24.4
Indonesia	17.0
Brazil	12.5
Mexico	10.8



Developed Countries IMR

Developed Countries	IMR
USA	5.5
Canada	4.4
UK	4.0
Germany	3.1
Japan	1.8



Descriptive Statistics

Using equations (1) and (2), the descriptive statistics of the IMR data are calculated and presented in Table 2.

Table 2. Descriptive statistics of IMR

Group	N	Mean	Variance
Developing countries	5	17.84	38.67
Developed countries	5	3.76	2.01

Two-Sample *t*-Test Between Developing and Developed Countries

The objective is to test whether there is a significant difference between the mean IMR of developing and developed countries.

The hypotheses are defined as:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

where μ_1 and μ_2 represent the population mean IMR of developing and developed countries, respectively.

Using equations (3) and (4), the pooled variance and the *t*-test statistic are calculated. The results are summarized in Table 3.

Table 3. Two-sample *t*-test results for IMR

Group	N	Mean	Variance	Pooled Variance	<i>t</i> -value	<i>p</i> -value
Developing	5	17.84	38.67	20.34	5.47	< 0.01
Developed	5	3.76	2.01			

Interpretation of Results

From table 3 above, the calculated value for *t* is 5.47. The critical value for the test at 5% significance level and 8 degrees of freedom is approximately 2.306. As the calculated value is greater than the critical value, the null hypothesis is rejected.

This outcome means that there exists a statistically significant difference between the average Infant Mortality Rates for developing and developed nations. The average IMR for developing countries is significantly greater than the average for developed countries.

5. Conclusion

Applying the two-sample t-test to IMR data proves that it is efficient in utilizing this statistical tool to analyze data on populations and their means for making important public health decisions and inference-making in global health, as it highlights that there is a significant difference between both groups on Infant Mortality Rate.

6. References

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